

CELLULAR AUTOMATA

A *cellular automaton*, or *polyautomaton*, is a theoretical model of a parallel computer, subject to various restrictions to make formal investigation of its computing powers tractable. All versions of the model share these properties: Each is an interconnection of identical cells, where a cell is a model of a computer with finite memory—i.e. a finite-state machine. Each cell computes an output from inputs it receives from a finite set of cells forming its neighborhood, and possibly from an external source.

All cells compute one output simultaneously and each cell computes an output at each tick of a clock, i.e. after each unit time step. The output of a cell is distributed to its neighborhood and possibly to an external receiver.

A version of the cellular automaton model exists for each set of choices in the following dichotomies: an infinite or a finite number of cells; a uniform interconnection scheme (all cells have neighborhoods of the same shape, e.g. that in Fig. 1) or a non-uniform scheme (Fig. 2); deterministic or non-deterministic cells (a

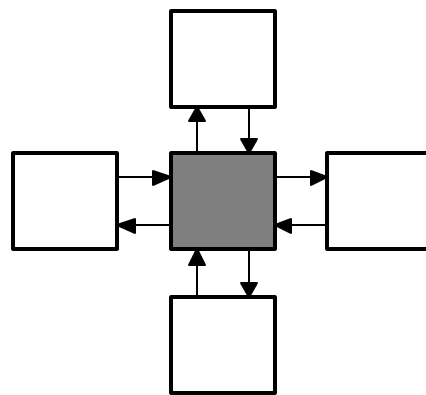


FIG. 1. A cell (hatched) and its neighborhood.

choice of exactly one output value at each unit time step or a set of several values); the absence or presence of an external input (output), and, in the case of an external input (output), it is connected to all cells or to only a subset; Moore-type or Mealy-type cells (unit time or zero time required, respectively, between inputs and the associated output); a static or dynamic interconnection scheme (neighborhood does or does not remain fixed in time). Some of the names associated with one or more of these versions are cellular automaton, cellular space, tessellation automaton, modular array, iterative automaton, intelligent graph, Lindenmayer system, and cellular network.

The first version of the cellular automaton, historically, was the cellular space obtained by selecting the first choice in each dichotomy above, but with no external input or output. It can be visualized in two dimensions as an infinite chessboard, each square representing a cell. It has been used to prove the existence of non-trivial self-reproducing machines, is capable of computing any

computable function with only three states per cell and the four nearest cells as the neighborhood (Fig. 1), and can exhibit *Garden of Eden configurations*; i.e. patterns of cell states at one time, which can never arise in a given cellular space except at time zero. If an external input is assumed distributed to each cell, then the cellular space becomes what is usually called a “tessellation” space.

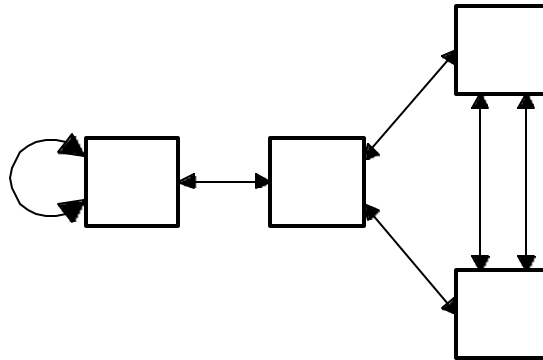


FIG. 2. A cellular automaton with non-uniform neighborhood.

The cellular automaton is obtained from the cellular space by admitting only a finite, connected set of cells on the chessboard (Fig. 3). A cell with a neighbor missing has a special boundary signal substituted instead. The cellular automaton is particularly useful as a pattern recognizer, where the pattern comprises the states of the cells at time zero, especially if non-deterministic cells are allowed. A famous problem for the (deterministic) cellular automaton, the Firing Squad problem, calls each cell a soldier with one of them as the general—i.e. all cells but one are “off” initially—and asks if all soldiers can begin firing simultaneously by going into the same state. The Firing Squad theorem, which solves this problem, guarantees a yes.

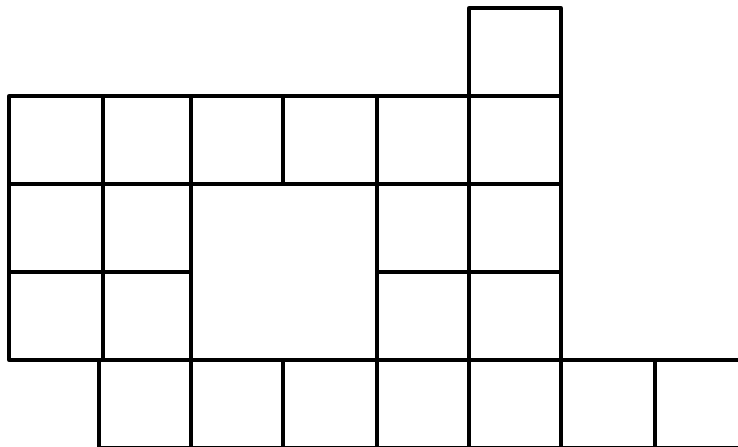


FIG. 3. A cellular automaton with uniform neighborhood of Fig. 1 assumed.

The Firing Squad theorem remains valid even when a non-uniform interconnection scheme is allowed. Thus, another version of the cellular automaton, the graphical cellular automaton (Fig. 2), requires only that the number of neighbors be fixed, not that they be in any fixed geometric relationship with a cell.

The final type of cellular automaton to be mentioned, the dynamic cellular automaton, or *Lindenmayer system* (or *L-system*), allows a cell to divide into children cells—regardless of the position of that cell in the initial array of cells—and allows the disappearance, or death, of cells. This version, with its dynamic interconnection scheme, is of interest to theoretical biologists as a model for the growth and development of living things. There are now in the 1990s many papers applying L-systems to the growth of realistic plants and trees (Plate 1).



Plate 1. “White Sands,” a plant-like object grown with an L-system.

The infinite chessboard cellular automaton model gained much public popularity in the 1970s as the so-called *Game of Life*. A resurgence of interest in the 1980s accompanied application of simple two-state, one-dimensional cellular automata (now often abbreviated CA) to fractals (*q.v.*) and dynamic chaos, rich new subjects that arose during that decade, propelled by inexpensive computer graphics (*q.v.*).

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